

Design of a Coherent Multimoded DIELECTRIC WAKEFIELD ACCELERATOR

J. Power & W. Gai

This AGN note determines the inner radius, **a**, the outer radius, **b**, and the dielectric constant, ϵ , for a dielectric lined cylindrical waveguide operating in what J. Hirshfield calls the “stimulated” mode. This method differs from the previous DWFA employed at AWA which drives only the fundamental longitudinal mode of the guide. We will call this scheme “coherent multimoded” DWFA since coherent implies all modes acting together to produce the net wakefield and multimoded implies more than one mode being driven.

Results are presented in the appendix for 4 cases: $\epsilon = [20, 36]$ and frequency spacing = [1.3 GHz, 2.6 GHz]. The detailed calculation shown here is only for the $\epsilon = 36$ and frequency spacing = 2.6 GHz case.

I) Define the waveguide (units are mks, except ϵ and μ which are relative)

$$\begin{aligned} \text{Inner radius} \quad a &:= 0.005 \text{ m} \\ \text{Outer radius} \quad b &:= 0.01467 \text{ m} \\ \text{Material Constants} \quad \epsilon &\equiv 36 \quad \mu := 1 \end{aligned}$$

II) Bunch Length

$$\sigma := 0.001 \text{ m}$$

III) Setup the boundary value problem and define the following functions

$$\begin{aligned} f1(x) &:= J_0(a \cdot x) \cdot Y_0(b \cdot x) - J_0(b \cdot x) \cdot Y_0(a \cdot x) \\ f2(x) &:= -J_1(a \cdot x) \cdot Y_0(b \cdot x) + J_0(b \cdot x) \cdot Y_1(a \cdot x) \\ f(x) &:= f2(x) - \frac{a}{(2 \cdot \epsilon)} \cdot x \cdot f1(x) \cdot (-1) \\ D(x) &:= \frac{d}{dx} f(x) \end{aligned}$$

IV) Find the first 100 resonant frequencies

a) Initial guesses in array x

$$\begin{aligned} imax &:= 100 \\ x_0 &:= 200 \\ i &:= 0 \dots imax \\ x_{i+1} &:= \text{if } [i < 80, 3.217 \cdot (i + 1) \cdot 10^2 + x_0, 3.22 \cdot (i + 1) \cdot 10^2 + x_0] \end{aligned}$$

b) Find the zeros of $f(x)$, store them in the array r and convert that into the resonant frequency

$$g(x) := \text{root}(f(x), x)$$

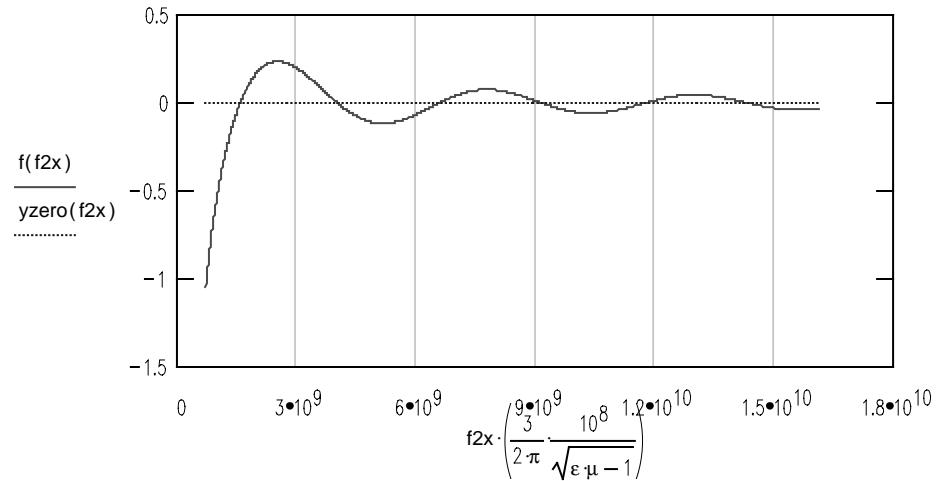
$$r_i := g(x_i)$$

$$\text{freq}_i := r_i \cdot \frac{3}{2\pi} \cdot \frac{10^8}{\sqrt{\epsilon \cdot \mu - 1}}$$

V) For amusement, plot the first few zeros of $f(x)$The zeros of $f(x)$ are the resonant frequencies

$$yzero(x) := 0$$

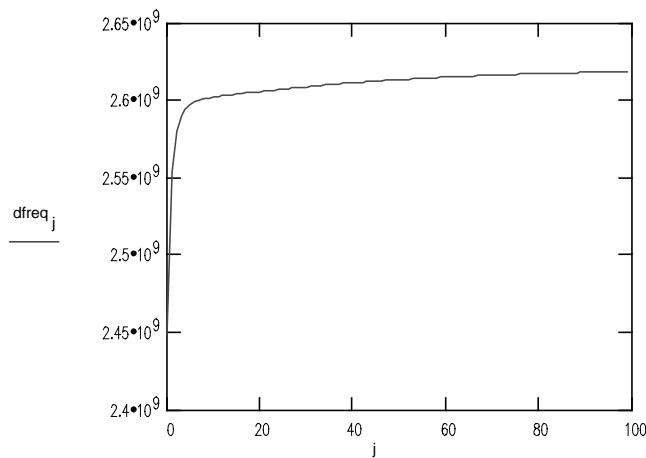
$$f2x := 90, 91.. 2000$$



VI) Calculate and Plot the Frequency Spacing of the Modes

$$j := 0, 1.. \text{imax} - 1$$

$$\text{dfreq}_j := \text{freq}_{j+1} - \text{freq}_j$$



	0
0	$2.449391 \cdot 10^9$
1	$2.554267 \cdot 10^9$
2	$2.580264 \cdot 10^9$
3	$2.590114 \cdot 10^9$
4	$2.594865 \cdot 10^9$
5	$2.597541 \cdot 10^9$
6	$2.599225 \cdot 10^9$
7	$2.600382 \cdot 10^9$
8	$2.601236 \cdot 10^9$

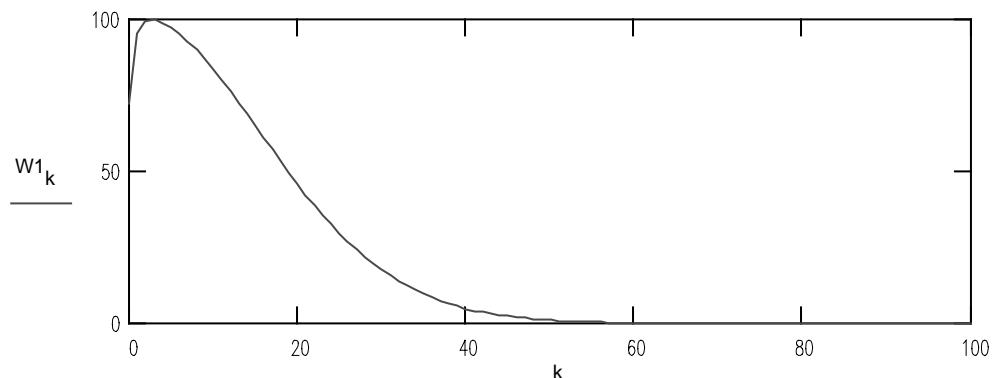
$$\text{avgdfreq} := \frac{\sum_{k=0}^{\text{imax}-1} \text{dfreq}_k}{\text{imax}}$$

$$\text{avgdfreq} = 2.609584 \cdot 10^9$$

VII) Plot the Power Spectrum for a Gaussian Beam up to the first 100 modes

$$W1_i := \frac{f1(r_i)}{D(r_i)} \cdot \exp \left[\frac{-(r_i \cdot \sigma)^2}{(\varepsilon \cdot \mu - 1) \cdot 2} \right]$$

$k := 0, 1.. \text{imax}$



*) note: The peak amplitude mode is at n = 3:

$$W1_3 = 99.770429 \blacksquare$$

VIII) Plot the Wake Potential (MeV/m/nC)

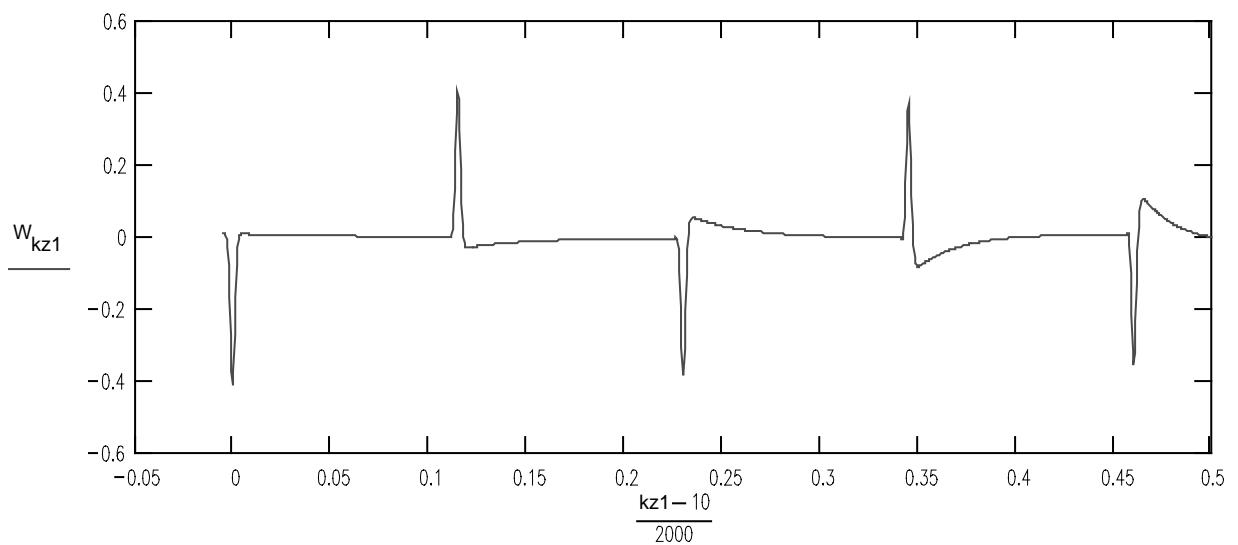
$$\eta := 4 \cdot \frac{1.44}{1.6} \cdot \frac{1}{a \cdot \epsilon \cdot 10^5}$$

$$W(z) := \left[\sum_i \frac{f_1(r_i)}{D(r_i)} \cdot \cos \left[\frac{r_i}{(\epsilon \cdot \mu - 1)^5} \cdot z \right] \cdot \exp \left[\frac{-(r_i \cdot \sigma)^2}{(\epsilon \cdot \mu - 1) \cdot 2} \right] \right] \cdot (-\eta) \cdot 1$$

$kz := 0, 1.. 1010$

$$W_{kz} := W \left[\frac{(kz - 10) \cdot 5}{10000} \right]$$

$kz1 := 0, 1.. 1010$



APPENDIX

A.1) Specification for the 4 tubes with error analysis. Target: $\lambda = 46.10 \text{ cm}$ ($f = 1.3 \text{ GHz}$) & 23.05 cm ($f = 2.6 \text{ GHz}$).

a (cm)	b (cm)	ϵ	λ (cm)	f (GHz)	Wakefield (MeV/m/nC)
0.5	3.133	20	46.10	1.3	.498
0.5	2.443	36	46.10	1.3	.403
0.5	1.811	20	23.05	2.6	.498
0.5	1.467	36	23.05	2.6	.403

Error check on ϵ

I) $v = 1.3 \text{ GHz}$

a (cm)	b (cm)	$\epsilon \pm 1$	λ (cm)	Wakefield (MeV/m/nC)
0.5	3.133	21	47.20 (+1.1)	.488
		19	44.80 (-1.3)	.505
0.5	2.443	37	46.75 (+0.65)	.397
		35	45.45 (-0.65)	.403

II) $v = 2.6 \text{ GHz}$ (double the error since we only send beams in at 46 cm)

a (cm)	b (cm)	$\epsilon \pm 1$	λ (cm)	Wakefield (MeV/m/nC)
0.5	1.811	21	23.65 (+0.60)	.488
		19	22.45 (-0.60)	.503
0.5	1.467	37	23.35 (+0.30)	.392
		35	22.70 (-0.35)	.403